

Downsampling-based multiple description image coding using optimal filtering

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Abstract. In this paper, a multiple description image coding scheme is proposed to facilitate the transmission of images over media with possible packet loss. The proposed method is based on finding the optimal reconstruction filter coefficients that will be used to reconstruct lost descriptions. For this purpose initially, the original image is downsampled and each subimage is coded using standard JPEG. These decoded images are then mapped to the original image size using the optimal filters. Multiple descriptions consist of coded down-sampled images and the corresponding optimal reconstruction filter coefficients. It is shown that the proposed method provided better results compared to standard interpolation filters (i.e., bicubic and bilinear). © 2008 SPIE and IS&T.
[DOI: 10.1117/1.2976420]

1 Introduction

The multiple description coding (MDC) approach is used for data transfer over packet networks, which are nowadays widespread. This approach fundamentally provides efficient transmission of multimedia data through these kinds of error-prone networks. MDC carries out this operation by coding and transmitting the original data using more than one bit stream and therefore reduces the influence of possible packet loss.

Various multimedia applications require data transmission over error-prone networks in which part of the data might not arrive at the receiver. Automatic repeat requests executed by the receiver are not possible for real-time data, such as voice and video, because these will cause long delays. The MDC approach enables reconstruction of data at an acceptable quality level in the case of possible packet losses. Original data are coded at the encoder into more than one packet (i.e., multiple descriptions) such that each one is self-decodable. When all descriptions reach the receiver, data are reconstructed at high quality; otherwise, acceptable quality data are still obtained. Nevertheless, coding efficiency is degraded due to redundancy introduced into descriptions.

MDC schemes can be grouped according to their computational complexity and redundancy insertion approaches. One of the first MDC methods makes use of multiple description scalar quantization (MDSQ),^{1,2} which uses overlapping quantization steps to enable redundancy. At the decoder the intersection of received quantization steps are used for inverse quantization. In Refs. 3 and 4 redundancy insertion is carried out using transforms referred to as pairwise correlating transform (PCT)-based approaches. These methods transform two input variables into two output variables and then encode the transformed variables. If one of the transformed variables is not received, then it can be estimated at a certain accuracy using the other variable. Polyphase downsampling (PD)-based MDC approaches have been proposed in Refs. 5–8. The first type of PD-based MDC approaches quantize input data at two different quantization levels after downsampling.^{5,6} The second type of PD-based MDC approaches perform oversampling, by making use of zero padding in the discrete cosine transform (DCT) domain in one or two dimensions, as presented in Refs. 7 and 8, respectively, before the descriptions are generated. In PD-based MDC approaches, if one of the descriptions is lost at the receiver, then the other samples are used to reconstruct the lost data. MDC redundancy is introduced using frame expansion in.^{9,10} Recently, wavelet-based MDC approaches have become popular.^{11–15} For example, the MDSQ approach is applied in the wavelet domain, and it is shown that MDC performance is increased compared to image domain coding. The method presented in Ref. 12 uses biorthogonal filter structures to construct wavelet descriptions having lower redundancy. It uses a fast converging iterative convex optimization approach to improve the quality at the receiver. A PD-based MDC approach is used in the wavelet domain in Ref. 13, and it is shown that the performance of this method is better than the MDSQ approach. MDC is directly used with a JPEG2000 coder in Ref. 14, where rate distortion characteristic of the input data is examined to introduce an adjustable level of redundancy. PCT is combined with wavelet transform-based image coding in Ref. 15, and it is shown that this outperforms MDC in the DCT domain.

Paper 07125RR received Jun. 27, 2007; revised manuscript received Apr. 30, 2008; accepted for publication May 2, 2008; published online Aug. 28, 2008.

1017-9909/2008/17(3)/033018/9/\$25.00 © 2008 SPIE and IS&T.

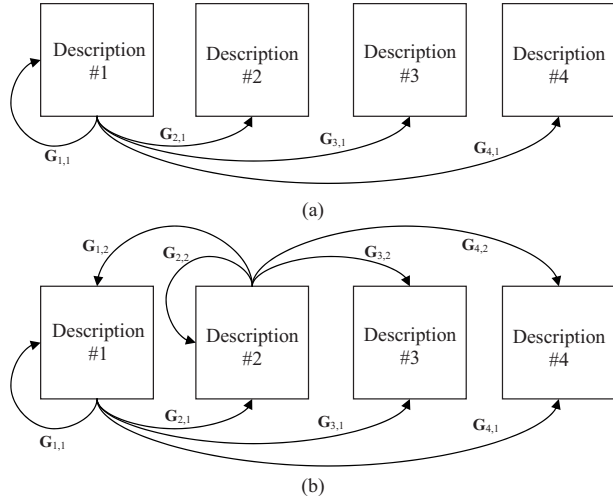


Fig. 1 Single-stage optimal filtering situation: (a) Only one (i.e., the first) description is received and (b) the first and second descriptions are received.

Downsampling-based image coding has recently been proposed for low bit-rate image coding.^{16,17} In this approach, images are downsampled prior to coding and up-sampled at the receiver to reconstruct the full resolution original image. In Ref. 16, an optimal filtering approach is used to reconstruct the full-resolution image. It is shown that this approach improves the performance significantly in JPEG coding. A similar approach is also applied to warped cosine transform-based image coding in Ref. 17. The optimal filtering approach is also used in Refs. 18 and 19 to improve efficiency of warped cosine transform-based image coding and H.264/AVC-based video coding, respectively.

In this paper, an optimal filtering-based MDC image coding approach that can improve the MDC performance is proposed. This method basically obtains filter coefficients by a least-squares approach, minimizing the difference between encoded and original data, for each description. A higher quality image can be reconstructed at the decoder, performing a filtering operation using the optimal filter coefficients that are included in the descriptions.

2 Proposed Approach

In the proposed approach, PD is used to obtain multiple descriptions. For simplicity, the proposed approach will be explained for four descriptions. In order to obtain the four PD-based multiple descriptions, images are initially down-sampled by a factor of two, in both the horizontal and vertical directions. Thus, four subimages are simply constructed as

$$I_1(i, j) = I(2i - 1, 2j - 1),$$

$$I_2(i, j) = I(2i, 2j - 1),$$

$$I_3(i, j) = I(2i - 1, 2j),$$

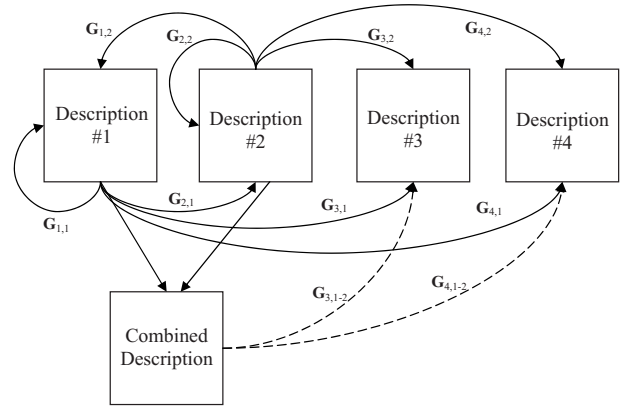


Fig. 2 Multistage optimal filtering.

$$I_4(i, j) = I(2i, 2j), \tag{1}$$

where (i, j) represents spatial position. Note that if w, h show the size of the original image then the size of the subimages (i.e., $I_1, I_2, I_3,$ and I_4) is $w/2, h/2$. After obtaining the subimages, each subimage can be encoded using the standard baseline JPEG at the desired quality level for MDC.

Assuming, \tilde{I}_n shows the encoded and decoded version of the subimage I_n , it is aimed to minimize the difference between the original and each encoded subimage using the optimal filtering approach. For example, it is shown in (2) how to obtain all subimages using optimal filtering of the encoded first subimage \tilde{I}_n with $n=1$, only. In this case, all four subimages ($\hat{I}_m, m=1, 2, 3, 4$) are reconstructed from the first encoded subimage using optimal filtering. Note that the asterisk shows two-dimensional (2-D) convolution. For each encoded subimage \tilde{I}_n ($n=1, 2, 3, 4$) four optimal filters $\mathbf{G}_{m,n}$ ($m=1, 2, 3, 4$) are defined to obtain all four reconstructed subimages at the receiver,

$$\hat{I}_{1,1} = \tilde{I}_1^* \mathbf{G}_{1,1},$$

$$\hat{I}_{2,1} = \tilde{I}_1^* \mathbf{G}_{2,1},$$

$$\hat{I}_{3,1} = \tilde{I}_1^* \mathbf{G}_{3,1},$$

$$\hat{I}_{4,1} = \tilde{I}_1^* \mathbf{G}_{4,1}. \tag{2}$$

The filtering operations for the other subimages are performed in a similar way to obtain all $\hat{I}_{m,n}$. The optimal reconstruction filters \mathbf{G} , have a dimension of $l \times l$ and are obtained in least-squares sense, as follows:

$$\min_{\mathbf{G}_{m,n}} \|I_m - \hat{I}_{m,n}\| = \min_{\mathbf{G}_{m,n}} \|I_m - \tilde{I}_n^* \mathbf{G}_{m,n}\| \quad m, n = 1, 2, 3, 4. \tag{3}$$

Note that the optimal filtering computations are accomplished over the subsampled images, and therefore, the difference between the subsampled original image (I_m) and

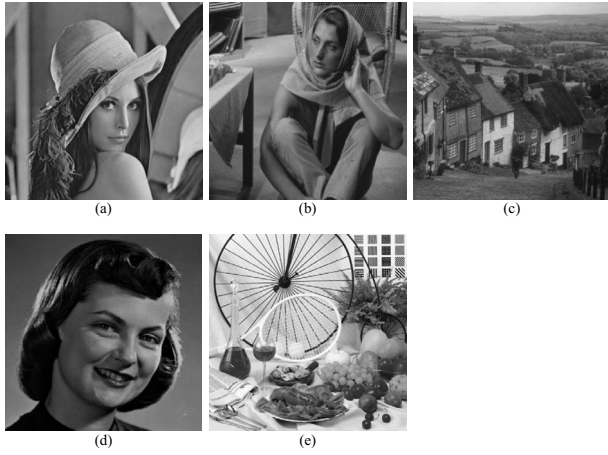


Fig. 3 Test images: (a) Lena, (b) Barbara, (c) Goldhill, (d) Girl, and (e) Bike.

the filtered encoded image ($\hat{I}_{m,n}$) is aimed to be minimized.

2.1 Iterative Preconditioned Conjugate Gradients Approach

Iterative preconditioned conjugate gradients²⁰ (IPCG) is used to solve (3). Actually, any other least-squares method

can be used instead of IPCG for this purpose, but IPCG is chosen because of its capability of solving large systems and providing fast convergence. IPCG requires less storage and is easier to implement compared to conventional methods. Furthermore, it provides an estimate of the solution at each step, which is better than the previous one. An unrolled implementation of IPCG is used in this work as described in Ref. 21.

Let us suppose that the vector \mathbf{g} is used to represent the filter kernel coefficients in row-stacked form for easiness, so that $\mathbf{g}(i+jl)=\mathbf{G}(i,j)$, $0 \leq i, j \leq l-1$. A vector \mathbf{b} is used to symbolize the original image values in row-stacked form so that $\mathbf{b}(i+jw/2)=I_m(i,j)$, $0 \leq i \leq w/2$, and $0 \leq j \leq h/2$. (where w and h represent the width and height of the original image), and the matrix \mathbf{A} with dimensions $(w \times h/4) \times l^2$ is

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_{0,0}^T \\ \mathbf{a}_{0,1}^T \\ \vdots \\ \mathbf{a}_{w/2,h/2}^T \end{bmatrix}, \tag{4}$$

where $\mathbf{a}_{i,j}^T$ is the row-stacked form of an $l \times l$ sized 2-D window centered around the pixel location (i,j) of \tilde{I}_n . Then,

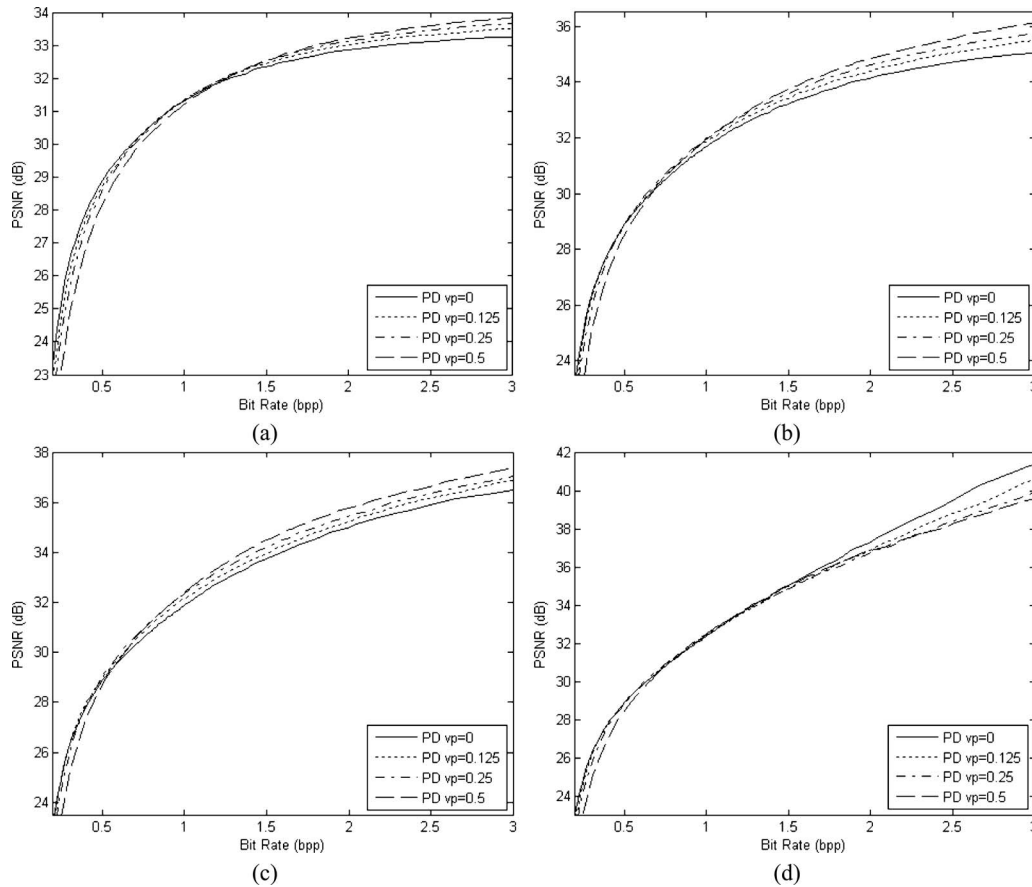


Fig. 4 Rate-distortion results for the Lena image in the case of different values of the oversampling parameter when different number of descriptions are received for the presented in Ref. 8: (a) One, (b) two (c) three, and (d) four description reception cases.

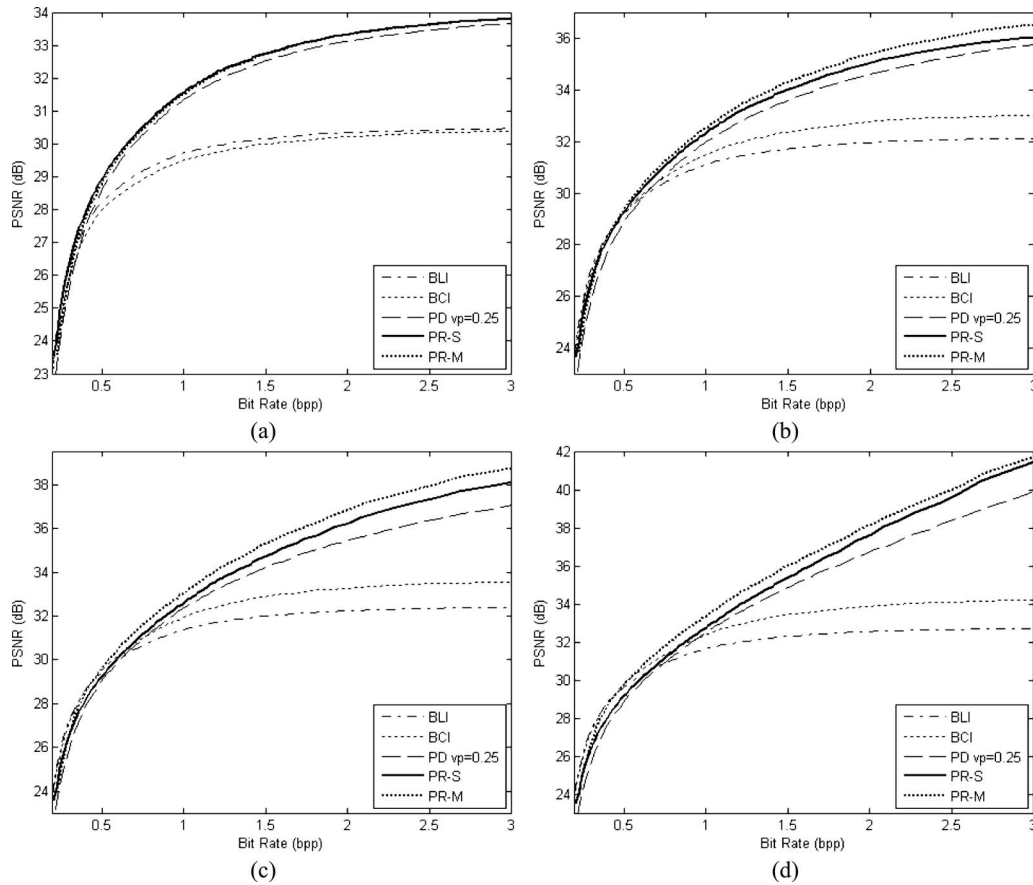


Fig. 5 Rate-distortion result for the Lena image: (a) One, (b) two, (c) three, and (d) four descriptions reception cases.

it is possible to express the system to be solved in the form of

$$\mathbf{A}\mathbf{g} = \mathbf{b}. \tag{5}$$

It is necessary to obtain a square coefficient matrix to find a solution for this equation system using preconditioned conjugate gradients. For this purpose, both sides of (5) can be multiplied with \mathbf{A}^T to make the matrix on the left-hand side of the equation a square matrix. Now, iterative preconditioned conjugate gradients can be utilized to solve this new equation system. Let us define $\bar{\mathbf{A}} = \mathbf{A}^T\mathbf{A}$ and $\bar{\mathbf{b}} = \mathbf{A}^T\mathbf{b}$ for simplicity, then it is possible to define the system to be solved in the form of

$$\bar{\mathbf{A}}\mathbf{g} = \bar{\mathbf{b}} \tag{6}$$

and the solution of this system will still give a least-squares solution when solved for \mathbf{g} . This system can now be solved using the unrolled IPCG implementation in Ref. 21.

2.2 Single-Stage Optimal Filtering

Optimal filter coefficients corresponding to all subimages are obtained as described in Section 2.1. Each description is formed using encoded subimage data and coefficients of the four corresponding optimal filters. Therefore, a total of 16 optimal filters are obtained at the encoder. If only one subimage is received at the decoder [for example, description

1, see Fig. 1(a)], the other subimages are obtained by simply filtering the received subimage with the corresponding optimal filters (i.e. $\mathbf{G}_{1,1}$, $\mathbf{G}_{2,1}$, $\mathbf{G}_{3,1}$, and $\mathbf{G}_{4,1}$ for the first description). When more than one description reaches the decoder, as for example depicted in Fig. 1(b), the lost descriptions are obtained via averaging the optimal filtering result of received descriptions, and the pixels of received descriptions are used from their actual description only (using the corresponding optimal filter $\mathbf{G}_{m,m}$) without averaging. After obtaining each subimage in the aforementioned way, these subimages are simply merged to obtain the full-resolution image. This approach is referred as single-stage optimal filtering.

2.3 Multistage Optimal Filtering

In Section 2.2, the reconstruction of the multiple descriptions using 16 optimal filters is explained. It is further possible to use additional optimal filters to construct a multistage optimal filtering framework for multiple description coding. In this case, we use the same approach to predict the coefficients of the reconstruction filters when two or three descriptions are received. That is, for example, when two descriptions are received, these will be combined and another optimal filtering stage will be used to improve the reconstruction accuracy using the combined image. This situation is depicted in Fig. 2, where only the first and second descriptions are received and the lost descriptions

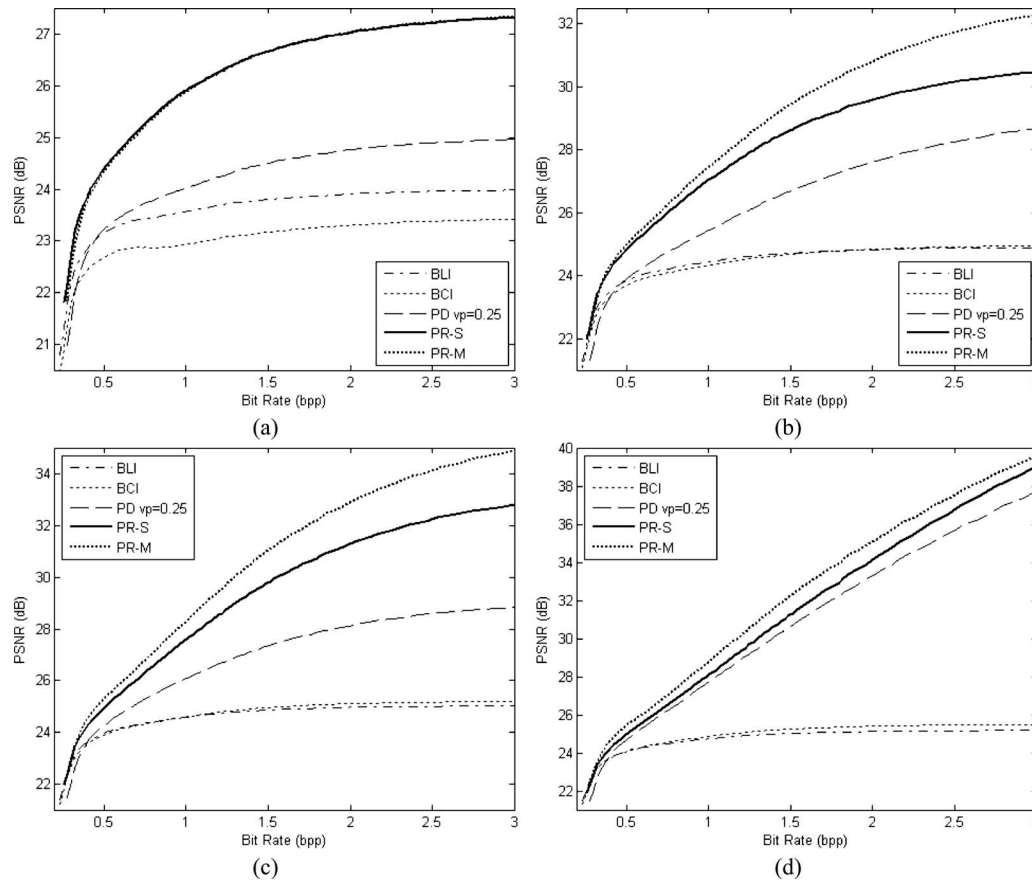


Fig. 6 Rate-distortion result for the Barbara image: (a) One, (b) two, (c) three, and (d) four descriptions reception cases.

are reconstructed using the combined image and a second stage of two optimal filters shown as $\mathbf{G}_{3,1-2}$ and $\mathbf{G}_{4,1-2}$. Such a multistage optimal filtering can be carried out to further improve the performance, which results in additionally six filters for the case where two descriptions are received and four filters for the case where three descriptions are received. The performance results of such a multistage approach will be investigated in the experimental results section.

2.4 Flexibility of the Proposed Method

Although the proposed method is explained for four descriptions, it is possible to use an arbitrary number of descriptions with the proposed scheme employing different subsampling patterns, such as quincunx subsampling.²² In this case, only the number of optimal filters will change. Furthermore, as it is stated in Ref. 8, in many practical situations only two descriptions are utilized. Therefore, we think that the four-description case is a good choice to demonstrate the effectiveness of the proposed method in practical situations.

The proposed method does not include any redundancy insertion scheme and is designed to be used with existing MDC methods that utilize a downsampling approach before multiple descriptions are generated. This flexibility of the proposed method provides an important useful feature. Within this scope, the proposed method can be regarded as a postprocessing approach.

The computational complexity of the proposed optimal filtering method is quite low, especially at the decoder side, because it only performs linear filtering operations using filter coefficients encoded into the bit stream. Computational timings are dependent on image size and are mostly not related to image content because the iterations take a very small amount of time. Most of the time is consumed at the matrix multiplication process before the iterations start. The filtering for a subimage of size 256×256 pixels requires 20 ms in a nonoptimized slow-working MatlabTM implementation using a Centrino 1.7 Gz processor on average. On the other hand, 750 ms is required for the computation of the coefficients of an optimal filter for the same image on average. It is obvious that the optimized implementation using a lower level programming language will decrease the computational load significantly. It is also possible to use separable filter kernels to further decrease the computational load at the expense of slight performance loss. Furthermore, it is shown in Ref. 19 that the optimal filtering approach can be executed at a speed of up to 20 fps for QCIF (quarter common intermediate format) image frames. From this point of view, it is clear that the optimized implementations will speed up the computational timings.

The introduced bit-rate overhead by the optimal filters is quite low. If the coefficients of the filters are represented in k -bit floatingpoint format without any compression, only $k \times l^2$ bits are required to encode them. If we consider an

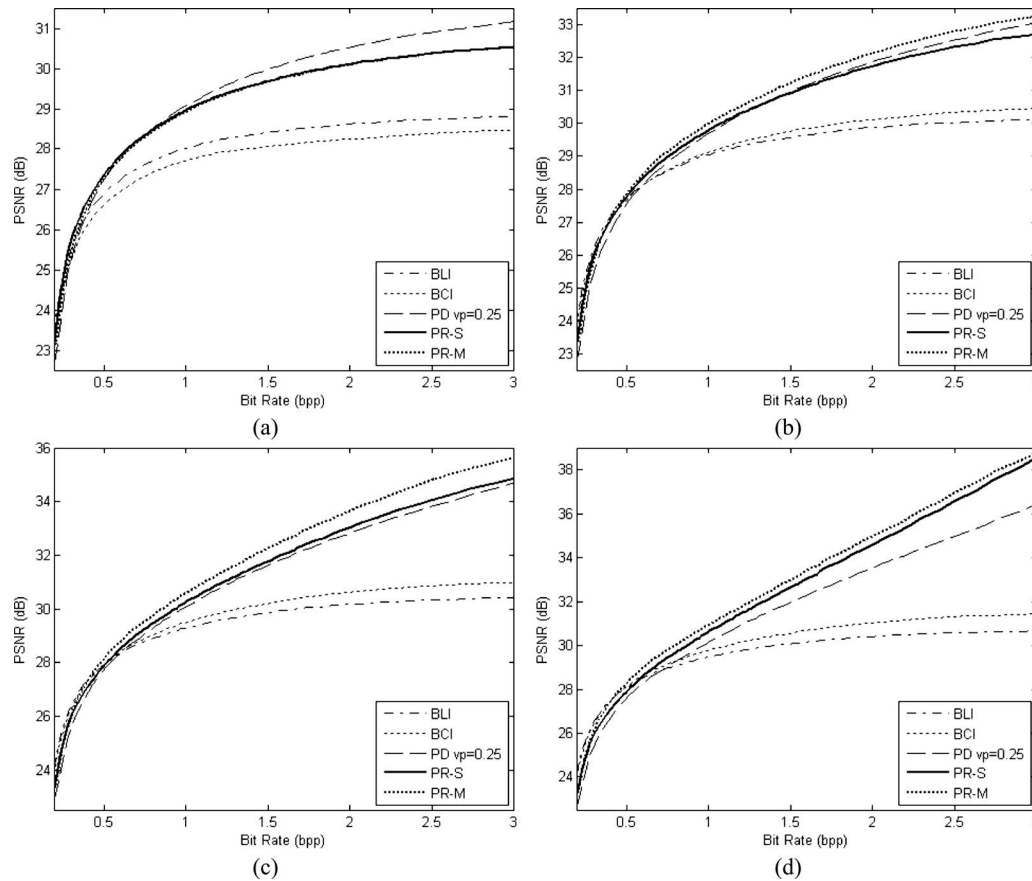


Fig. 7 Rate-distortion result for the Goldhill image: (a) One, (b) two, (c) three, and (d) four descriptions reception cases.

image size of 512×512 pixels and a filter size of 5×5 with $k=16$, the introduced overhead for one filter is only 0.0015 bit/pixel. A total of 16 filters are required for the single-stage case, whereas ten additional filters are necessary for the multistage case. From these results, it is clear that the overall bit-rate overhead is small. This overhead is nonetheless included in the results presented in this paper.

3 Experimental Results

The Lena, Barbara, Peppers, Boats, Goldhill, Girl, and Bike images of size 512×512 pixels are used to evaluate the performance of the proposed method. Small versions of these images are given in Fig. 3. Filter sizes are chosen as 5×5 because this size balances computational complexity and estimation performance. Conventional bilinear interpolation (BLI) and bicubic interpolation (BCI) approaches and one of the latest PD-based approaches as presented in Ref. 8 are also employed for comparison. The PD approach in Ref. 8 uses one-dimensional oversampling that is reported to outperform the two-dimensional oversampling case of Ref. 7. The approach in Ref. 8 also has a similar MD structure as the proposed approach, which is the reason for the method presented in Ref. 8 to be used for comparison. Note that the method in Ref. 8 is actually reported using JPEG2000, but results in this paper are provided for JPEG to enable a fair comparison.

Typical bit-rate versus distortion plots are used for objective comparison of the proposed method. We employed

the peak signal-to-noise ratio (PSNR) as the distortion measure. The method presented in Ref. 8 is used for comparison. This method has an oversampling ratio parameter that influences the performance of the method. In order to find a proper value for this parameter, a test on the Lena image is carried out. Figure 4 shows rate-distortion results for the Lena image in the case of different values of the oversampling parameter when one, two, three, and four descriptions are received. The performance of the method proposed in Ref. 8 improves at low bit rates when the oversampling ratio is low. If the oversampling ratio is increased, then its performance gets better at high bit rates. This is an expected situation because excessive oversampling is not useful at low bit rates because the bit budget is limited and oversampling cannot compensate for the effect of coarse quantization. On the basis of these experiments, the parameter (vp) is to 0.25 in the following results since this value provides good balance between lower and higher bit-rate ranges.

Figures 5–9 show the performance results for the Lena, Barbara, Goldhill, Girl, and Bike images, respectively, for different description cases. In these figures, BLI, BCI, PD, PR-S, and PR-M show bilinear interpolation, bicubic interpolation, the method presented in Ref. 8, the proposed single-stage optimal filtering scheme, and the proposed multistage optimal filtering scheme, respectively. Note that all overheads related to optimal filtering coefficients are taken into account in all these figures.

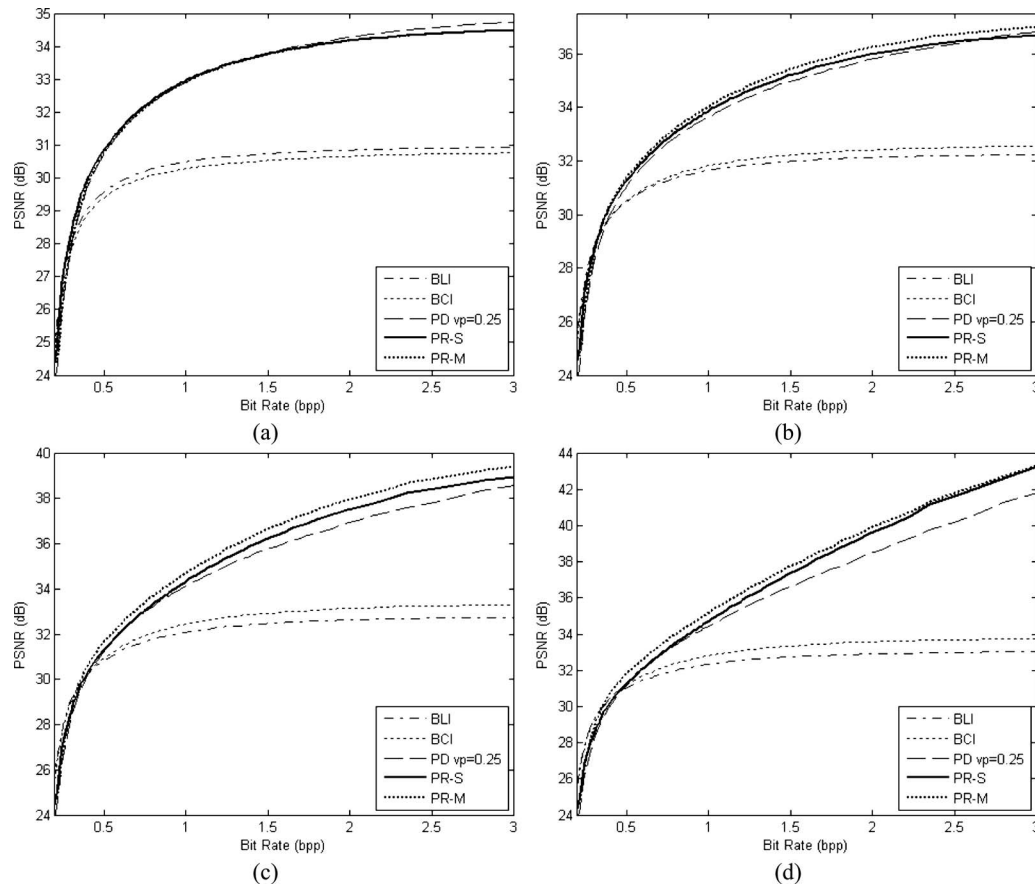


Fig. 8 Rate-distortion result for the Girl image: (a) One, (b) two, (c) three, and (d) four descriptions reception cases.

Rate-distortion results for the Lena image, which contains both low and high spatial detailed regions, are given in Fig. 5. As is obvious from these results, the proposed optimal filtering-based approaches provide better results. BLI and BCI upsampling approaches cannot show reasonable results because they use simple interpolation schemes, which are not very appropriate for MDC. The PD-based approach presented in Ref. 8 provides good results, especially when few descriptions are received compared to the proposed method. But the proposed method significantly outperforms this method when more descriptions reach the receiver. Note that the proposed PR-M provides better results with respect to PR-S thanks to the additional multi-stage filtering.

Figure 6 shows rate-distortion results for the Barbara image, which contains high spatial detail. It is seen from Fig. 6 that the proposed method significantly outperforms the other methods for all description cases. Although the difference in performance is usually large, only in the case of all four descriptions, the results of Ref. 8 get close to the proposed methods. The main reason might be the low-performance bidirectional linear estimator used in Ref. 8 which might not be able to obtain the lost spatial detail properly. A similar situation is valid for the BLI and BCI cases.

In Fig. 7, rate-distortion results for the Goldhill image, which has smooth areas, is presented. In this case, when only one description is received, the method in Ref. 8 pro-

vides better performance than our methods for ranges above 0.8 bits/pixel. When the number of received descriptions is increased, the performances of the proposed methods get better. The main reasons for the good performance of Ref. 8 for one description and high bit rates are as follows: (i) the used oversampling approach in Ref. 8 estimates smooth areas more efficiently and (ii) the oversampling overhead is less effective at higher bitrates. On the basis of these facts, the method in Ref. 8 provides good results for images, which include mostly smooth regions if only a small number of descriptions are received.

In Fig. 8, rate-distortion results for the Girl image, which has smooth and detailed areas, are given. Again, the performances of the proposed methods are the best except at high bit rates in the case of one description.

Rate-distortion results for the Bike image are given in Fig. 9. This image has extremely high spatial detail, and it is seen that the PSNR results of all approaches are very low compared to the other images. In this case, the method proposed in Ref. 8 provides the best performance for the single-description case, the performances are similar for the two-description case, while the performances of the proposed approaches are better for three and four descriptions.

When we evaluate the overall performance of BLI and BCI for the other images, it is clear that BLI provides better performance in the case of one description. However, when

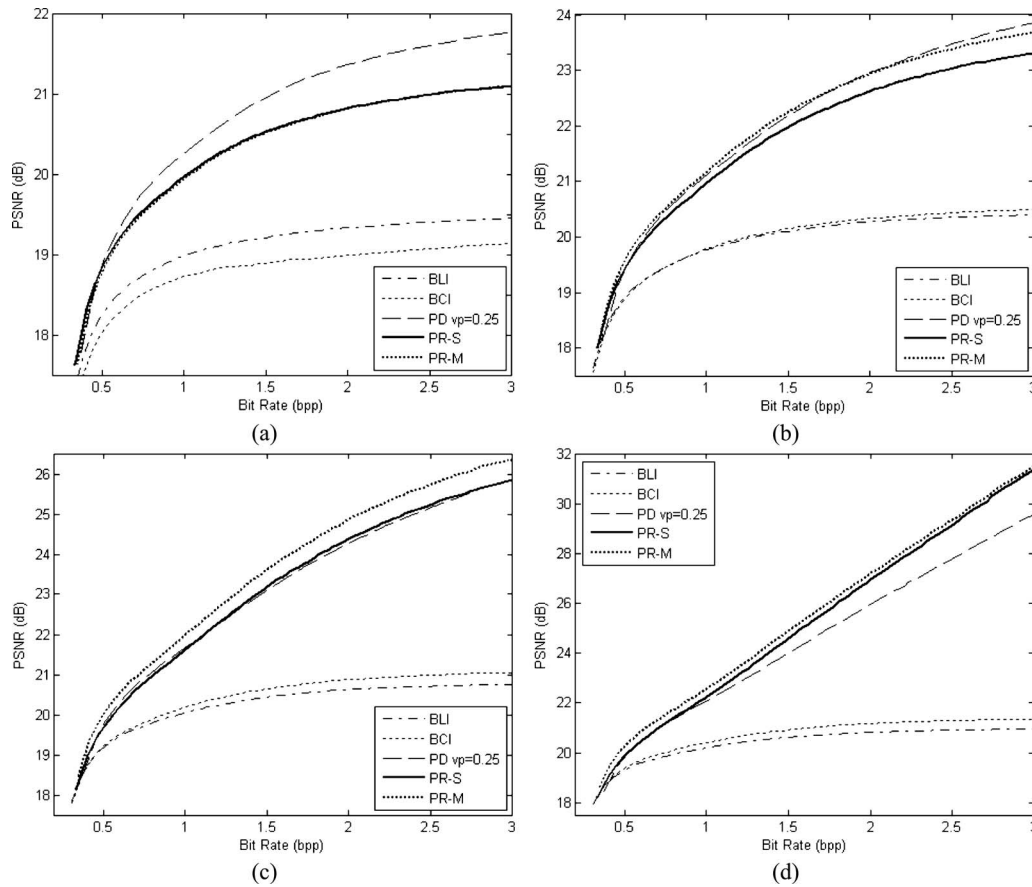


Fig. 9 Rate-distortion result for the Bike image: (a) One, (b) two, (c) three, and (d) four descriptions reception cases.

more descriptions are received the performance of BCI gets better. It is worth noting a significant difference for the one-description case for the Barbara image.

An additional experiment is carried out for the evaluation of the proposed method if only two descriptions are generated because the method in Ref. 8 is mainly presented for the two-description case. For this purpose, the Barbara and Goldhill images are used to show the performance of

our approach in the limit cases (i.e., the best and worst situations), and results are given in Fig. 10. As is seen from these results, when one description is received, the approach proposed in this paper provides better results, overall, for the Barbara image and at the low bit-rate range of the Goldhill image. On the other hand, in the high bit-rate range of the Goldhill image the method in Ref. 8 gives better results. If both descriptions reach the receiver, then

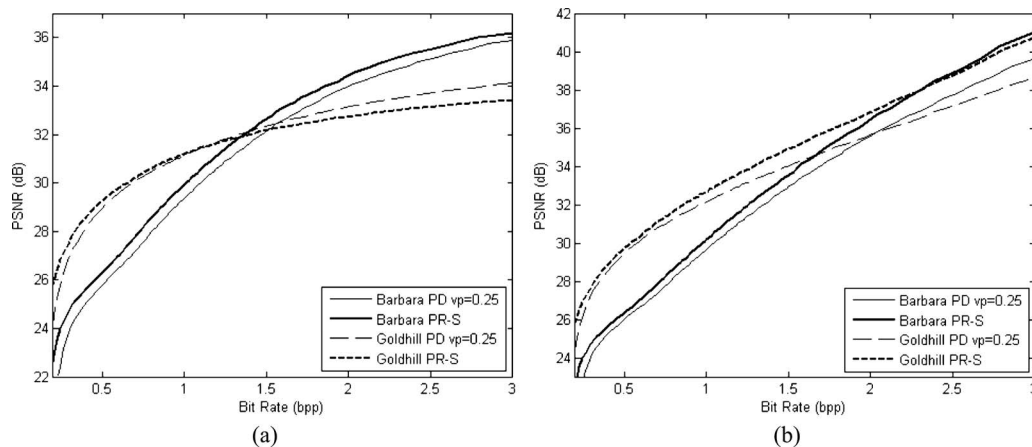


Fig. 10 Rate-distortion result when only two descriptions are generated: (a) One and (b) two descriptions reception cases.

the proposed method gives better results for both images. On the basis of these experiments, it is clear that the proposed method is more efficient.

4 Conclusions

A novel MDC scheme based on optimal filtering is presented in this paper. The image is downsampled prior to encoding, and coefficients of optimal filters are obtained for each subimage, minimizing the difference between the original subimages and their encoded versions. Experimental results show that the proposed approach outperforms classical interpolation approaches, such as bilinear and bicubic interpolation. Furthermore, the proposed approach mostly provides better results when compared to a recently proposed poly-phase downsampling MDC approach.

Acknowledgments

This work was supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under Grant No. EEEAG/107E155. The authors thank the anonymous reviewers for comments that improved the clarity of the paper. The authors also thank T. Tillo, Ç. Ateş, and Y. Ürgün for their support at the implementation and adaptation stages of the MDC method presented in Ref. 8.

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